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Spectroscopy of Heavy Mesons Expanded in $1/m_Q$

by

Takayuki Matsuki* Hirotoshi Watanabe† and Syunya Inoue‡

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Operating just once with the naive Foldy-Wouthuysen-Tani transformation on the Schrödinger equation for $Q\bar{q}$ bound states described by the Hamiltonian which includes Coulomb-like as well as confining scalar potentials, we have calculated the heavy meson spectrum of D, D^*, B, and B^*. Based on the formulation recently proposed, their masses and wave functions are expanded in $1/m_Q$, with a heavy quark mass m_Q, up to the first order. The lowest order equation is examined carefully to obtain a complete set of eigenfunctions for the Schrödinger equation.

Introduction

Hadrons are composed of quarks and anti-quarks and are considered to be governed by Quantum Chromodynamics, at least in principle. Since QCD describes a strong coupling interaction, the perturbative calculation of physical properties of hadrons is not so reliable other than the deep inelastic region due to asymptotic freedom and hence other methods like lattice gauge theory have been developed to take into account nonperturbative effects. However, the situation changed dramatically when it was discovered that heavy mesons, composed of heavy quark Q and light anti-quark $\bar{q}$, can be systematically expanded in $1/m_Q$ with a heavy quark mass, m_Q.

This theory, HQET (Heavy Quark Effective Theory),[1] is applied to many aspects of high energy theories and many kinds of physical quantities of QCD which can be perturbatively calculated in $1/m_Q$. Especially those regarding B meson physics, e.g., the lowest order form factor (which is now called Isgur-Wise function) of semileptonic weak deacy processes $B \to D\ell\nu$ and the Kobayashi-Maskawa matrix element V_{cb}, have been calculated by many people.[3] However, applications of HQET to higher order perturbative calculations are limited only to obtain forms of higher order operators, and their coefficients should be obtained so that results be fitted with experimental data.[2] This is because most of the calculations based on HQET do not introduce heavy meson wave functions and hence there is no way to determine those coefficients within the model.

In the former paper [4], using the Foldy-Wouthuysen-Tani transformation [5] we have developed a formulation so that the Schrödinger equation for a $Q\bar{q}$ bound state can be expanded in terms of $1/m_Q$, that is, the resulting eigenvalues as well as wave functions are obtained order by order in $1/m_Q$. In this paper, as one of the applications of our formulation, we will calculate the heavy meson spectrum of D, D^*, B, and B^*. In order to do so, we would like to start by introducing phenomenological dynamics, i.e., assuming Coulomb-like vector and confining scalar potentials to $Q\bar{q}$ bound states (heavy mesons), expand the Hamiltonian in $1/m_Q$ and then perturbatively solve the Schrödinger equation in $1/m_Q$. The angular part of the lowest order wave function is exactly solved. After extracting the asymptotic forms of the lowest order wave function at both $r \to 0$ and $r \to \infty$ and adopting the variational method, we numerically obtain the radial part of the trial wave function which is expanded in polynomials of the radial variable r. Then fitting the smallest eigenvalues of a Hamiltonian with masses of D and D^* mesons, a strong coupling α_s and other parameters included in the scalar and vector potentials are determined uniquely. Using parameters

*Laboratory for Information science, College of Home Economics
†Laboratory for Physics, College of Home Economics
‡Laboratory for Information science, College of Liberal Arts

obtained this way, other mass levels are calculated and fitted with experimental data for D/B mesons at the second order of perturbation. Meson wave functions obtained thereby and expanded in $1/m_Q$ may be used to calculate ordinary form factors as well as Isgur-Wise functions for semileptonic weak decay processes.

All the above calculations are calculated up to $1/m^2_Q$ and analyzed order by order in $1/m_Q$ to determine parameters as well as to compare with results of Heavy Quark Effective Theory. The final goal of this paper is to obtain higher orders of Isgur-Wise functions, decay constants of heavy mesons, and the Kobayashi-Maskawa matrix element, V_{cb}, by using wave functions of heavy mesons obtained after calculating heavy meson spectrum.
Below we will first give the formulation of this study and next will give a qualitative discussion on the results obtained.

II. HAMILTONIAN

The hamiltonian density for our problem is given by

$$\mathcal{H}_0 = \int dx^3 \left[q^{\dagger c}(x)\, (\vec{\alpha}_q \cdot \vec{p}_q + \beta_q m_q)\, q^c(x) + Q^\dagger(x)\, (\vec{\alpha}_Q \cdot \vec{p}_Q + \beta_Q m_Q)\, Q(x) \right], \tag{1}$$

$$\mathcal{H}_{\rm int} = \int\int dx^3 dx'^3\, \bar{q}^c(x)\, O_i\, q^c(x)\, V_i(x - x')\, \bar{Q}(x')\, O_i\, Q(x'), \tag{2}$$

where we consider only a scalar confining potential, $O_S=1$, $V_S=S(r)$, and a vector potential, $O_V=\gamma_\mu$, $V_V=V(r)$, with a relative radial variable r, which we think the best choice to phenomenologically describe the meson mass levels. The state of $Q\bar{q}$ is defined by

$$|\psi\rangle = \int d^3x \int d^3y\, \psi_{\alpha\beta}(x - y)\, q^{c\,\dagger}_\alpha(x)\, Q^\dagger_\beta(y)\, |0\rangle, \tag{3}$$

where $q^c(x)$ is a charge conjugate field of a light quark q and the conjugate state of $Q\bar{q}$ by $\langle\Psi| = |\Psi\rangle^\dagger$ with $\langle 0| = |0\rangle^\dagger$ From these definitions, we obtain the Schrödinger equation as

$$H\,\psi = (m_Q + \tilde{E})\,\psi, \tag{4}$$

where the bound state mass, E, is split into two parts, m_Q and $\tilde{E}$ ($E=m_Q+\tilde{E}$), so that it expresses the fact that the heavy quark mass is dominant in the bound state, $Q\bar{q}$, and Ψ is nothing but the wave function which appears in the rhs of Eq. (3).

Operating with the FWT transformation and a charge conjugation operator only on a heavy quark sector in this equation at the center of the mass system of a bound state, one can modify the Schrödinger equation as,

$$(H_{FWT} - m_Q) \otimes \psi_{FWT} = \tilde{E}\,\psi_{FWT}, \tag{5}$$

where a notation $\otimes$ is introduced to denote that gamma matrices of a light anti-quark is multiplied from left while those of a heavy quark from right and

$$H_{FWT} = U_c\, U_{FWT}(p'_Q)\, H\, U^{-1}_{FWT}(p_Q)\, U_c^{-1}, \quad \psi_{FWT} = U_c\, U_{FWT}(p_Q)\, \psi, \tag{6}$$

$$U_{FWT}(p) = \exp\left(W(p)\, \vec{\gamma}_Q \cdot \hat{p}\right) = \cos W + \vec{\gamma}_Q \cdot \hat{p}\, \sin W, \tag{7}$$

$$\hat{p} = \frac{\vec{p}}{p}, \quad \tan W(p) = \frac{p}{m + E}, \tag{8}$$

$$U_c = \gamma^0_Q \gamma^2_Q. \tag{9}$$

As described first in this section, interaction terms are given by a confining scalar potential and a Coulomb vector potential with transverse interaction [6], [7] and a total Hamiltonian is given by

$$H = (\vec{\alpha}_q \cdot \vec{p}_q + \beta_q m_q) + (\vec{\alpha}_Q \cdot \vec{p}_Q + \beta_Q m_Q) + \beta_q \beta_Q S + \left\{1 - \frac{1}{2}\left[\vec{\alpha}_q \cdot \vec{\alpha}_Q + (\vec{\alpha}_q \cdot \vec{n})(\vec{\alpha}_Q \cdot \vec{n})\right]\right\} V, \tag{10}$$

where scalar and vector potentials are given by

$$S(r) = \frac{r}{a^2} + b, \quad V(r) = -\frac{4}{3}\frac{\alpha_s}{r} \quad \text{and} \quad \vec{n} = \frac{\vec{r}}{r}. \tag{11}$$

The transformed Hamiltonian is expanded in $1/m_Q$ and is given by

$$H_{FWT} - m_Q = H_{-1} + H_0 + H_1 + H_2 + \cdots, \tag{12}$$

where

$$H_{-1} = -(1 + \beta_Q)\, m_Q, \tag{13}$$

$$H_0 = \vec{\alpha}_q \cdot \vec{p} + \beta_q m_q - \beta_q \beta_Q S + \left\{1 + \frac{1}{2}\left[\vec{\alpha}_q \cdot \vec{\alpha}_Q + (\vec{\alpha}_q \cdot \vec{n})(\vec{\alpha}_Q \cdot \vec{n})\right]\right\} V, \tag{14}$$

$$H_1 = -\frac{1}{2m_Q}\beta_Q \vec{p}^{\,2} + \frac{1}{m_Q}\vec{\alpha}_Q \cdot \left(\vec{p} + \frac{1}{2}\vec{q}\right) S + \frac{1}{2m_Q}\vec{\gamma}_Q \cdot \vec{q} V - \frac{1}{2m_Q}\left[\vec{\alpha}_q + (\vec{\alpha}_q \cdot \vec{n})\vec{n}\right] \cdot \left[\beta_Q\left(\vec{p} + \frac{1}{2}\vec{q}\right) + i\vec{q} \times \beta_Q \vec{\Sigma}_Q\right] V, \tag{15}$$

$$H_2 = \frac{1}{2m_Q^2}\beta_q\beta_Q\left(\vec{p} + \frac{1}{2}\vec{q}\right)^2 S - \frac{i}{4m_Q^2}\vec{q} \times \vec{p} \cdot \beta_q\beta_Q \vec{\Sigma}_Q S - \frac{1}{8m_Q^2}\vec{q}^{\,2} V - \frac{i}{4m_Q^2}\vec{q} \times \vec{p} \cdot \vec{\Sigma}_Q V - \frac{1}{8m_Q^2}\left[\vec{\alpha}_q + (\vec{\alpha}_q \cdot \vec{n})\vec{n}\right] \cdot \left[(\vec{p} + \vec{q})(\vec{\alpha}_Q \cdot \vec{p}) + \vec{p}(\vec{\alpha}_Q \cdot (\vec{p} + \vec{q})) + i\vec{q} \times \vec{p}\gamma_Q^5\right] V, \tag{16}$$

Here Hi stands for the i-th order expanded Hamiltonian terms and since a bound state is at rest,

$$\vec{p} = \vec{p}_q = -\vec{p}_Q, \quad \vec{p}\,' = \vec{p}_q\,' = -\vec{p}_Q\,', \quad \vec{q} = \vec{p}\,' - \vec{p}, \tag{17}$$

are defined, where primed quantities are final momenta. Details of the derivation of equations in this section are given in the paper [8].

III. PERTURBATION

Using the Hamiltonian obtained in the last section, we give in this section the Schrödinger equation order by order in $1/m_Q$. Details of the derivation in this section will be given in a future paper[8]. First we introduce projection operators:

$$\Lambda_\pm = \frac{1 \pm \beta_Q}{2}, \tag{18}$$

which correspond to positive-/negative-energy projection operators for a heavy quark sector at the rest frame of a bound state. These are given by $(1 \pm \psi)/2$ in the moving frame of a bound state with v^μ the four-velocity of a bound state. Then we expand the mass and wave function of a bound state in $1/m_Q$ as

$$\tilde{E} = E_0^\ell + E_1^\ell + E_2^\ell + \dots, \tag{19}$$

$$\psi_{FWT} = \psi_0^\ell + \psi_1^\ell + \psi_2^\ell + \dots, \tag{20}$$

where ℓ stands for a set of quantum numbers that distinguish independent eigenfunctions of the lowest order Schrödinger equation, and a subscript i of E_i^ℓ and Ψ_i^ℓ for the order of $1/m_Q$.

A. Leading order

The leading order Schrödinger equation in $1/m_Q$ gives

$$\psi_0^\ell = \Lambda_- \otimes \psi_0^\ell, \tag{21}$$

whose explicit form is given by

$$\psi_0^\ell = \Psi_\ell^+ = \left(0 \quad \Psi_{jm}^k(\vec{r})\right), \tag{22}$$

with

$$\Psi_{jm}^k(\vec{r}) = \frac{1}{r}\begin{pmatrix} u_k(r) \\ -i\, v_k(r)\,(\vec{\sigma} \cdot \vec{n}) \end{pmatrix} y_{jm}^k(\Omega), \tag{23}$$

where j is a total angular momentum of a meson, m is its z component, k is a quantum number which takes only values, $k = \pm j, \pm(j+1)$ and $\neq 0$, and $u_k(r)$ and $v_k(r)$ are polynomials of a radial variable r. $y_{jm}^k(\Omega)$ are functions of angles and spinors of a total angular momentum, $\vec{j} = \vec{l} + \vec{s}_q + \vec{s}_Q$. The operator for the quantum number k is given by $-\beta_q\left(\vec{\Sigma}_q \cdot \vec{l} + 1\right)$ when it operates on $\left(0\ \Psi_{jm}^k(\vec{r})\right)$.

Note that since charge conjugation is operated on the heavy quark sector the Λ_- projection operator appears in Eq. (21), i.e., positive components of Q correpond to negative components of $U^c Q$.

B. 0-th order

The 0-th order equations are given by

$$\left[\vec{\alpha}_q \cdot \vec{p} + \beta_q (m_q + S) + V\right] \otimes \psi_0^\ell = E_0^\ell \psi_0^\ell, \tag{24}$$

$$-2m_Q\Lambda_+ \otimes \psi_1^\ell + \tfrac{1}{2}\Lambda_-\left[\vec{\alpha}_q \cdot \vec{\alpha}_Q + (\vec{\alpha}_q \cdot \vec{n})(\vec{\alpha}_Q \cdot \vec{n})\right] V \otimes \psi_0^\ell = 0. \tag{25}$$

Eq.(24) gives the lowest non-trivial Schrödinger equation with a solution given by Eq.(22) and $\vec{n}$ is defined in Eq.(11). Λ_+ components of wave functions can be expanded in terms of the eigenfunctions,

$$\Psi_\ell^- = \left(\Psi_{jm}^k(\vec{r}) \quad 0\right). \tag{26}$$

Expanding $\Lambda_+ \otimes \Psi_1^\ell$ in terms of this set of eigenfunctions, one can obtain the solution as

$$\Lambda_+ \otimes \psi_1^\ell = \sum_{\ell'} c_{1-}^{\ell\ell'} \Psi_{\ell'}^-, \qquad (27)$$

with the coefficients,

$$c_{1-}^{\ell\ell'} = \frac{1}{4m_Q} \left\langle \Psi_{\ell'}^- \middle| \left(\vec{\alpha}_q \cdot \vec{\alpha}_Q + (\vec{\alpha}_q \cdot \vec{n})(\vec{\alpha}_Q \cdot \vec{n})\right) V \middle| \Psi_\ell^+ \right\rangle. \qquad (28)$$

Here the inner product is defined to be

$$\left\langle \Psi_{\ell'}^- \middle| O \middle| \Psi_\ell^+ \right\rangle = \int d^3r \, \mathrm{tr}\left(\Psi_{\ell'}^{-\dagger} \left(O \otimes \Psi_\ell^+\right)\right), \qquad (29)$$

and the 0-th order wave functions are normalized to be 1,

$$\left\langle \Psi_\ell^\alpha \middle| \Psi_{\ell'}^\beta \right\rangle = \delta_{\ell\ell'} \delta^{\alpha\beta} \quad \text{for} \quad \alpha, \beta = + \text{ or } -. \qquad (30)$$

C. 1st order

The 1st order equation is given by

$$-2m_Q \Lambda_+ \otimes \psi_2^\ell + H_0 \otimes \psi_1^\ell + H_1 \otimes \psi_0^\ell = E_0^\ell \psi_1^\ell + E_1^\ell \psi_0^\ell. \qquad (31)$$

Multiplying projection operators $\Lambda\pm$ from right with the above equation, and expanding Ψ_1^k in terms of Ψ_k^+ and Ψ_k^- as

$$\psi_1^\ell = \sum_{\ell'} \left(c_{1+}^{\ell\ell'} \Psi_{\ell'}^+ + c_{1-}^{\ell\ell'} \Psi_{\ell'}^-\right), \qquad (32)$$

one obtains

$$E_1^\ell = \sum_{\ell'} c_{1-}^{\ell\ell'} \left\langle \Psi_\ell^+ \middle| \Lambda_+ H_0 \Lambda_- \middle| \Psi_{\ell'}^- \right\rangle + \left\langle \Psi_\ell^+ \middle| \Lambda_- H_1 \Lambda_- \middle| \Psi_\ell^+ \right\rangle, \qquad (33)$$

which gives the first order perturbation correction to the mass when one calculates matrix elements of the rhs among eigenfunctions and

$$c_{1+}^{\ell k} = \frac{1}{E_0^\ell - E_0^k} \left[\sum_{\ell'} c_{1-}^{\ell\ell'} \left\langle \Psi_k^+ \middle| \Lambda_+ H_0 \Lambda_- \middle| \Psi_{\ell'}^- \right\rangle + \left\langle \Psi_k^+ \middle| \Lambda_- H_1 \Lambda_- \middle| \Psi_\ell^+ \right\rangle \right], \quad \text{for } k \neq \ell \qquad (34)$$

$$c_{1+}^{kk} = 0. \qquad (35)$$

This completes the solution for Ψ_1^ℓ since $\Lambda_- \Psi_1^\ell$, or $C_{1-}^{\ell\ell'}$ is obtained in the last subsection. Here we have used the normaliztion for the total wave function, Ψ^ℓ, as

$$\left\langle \psi^\ell \middle| \psi^{\ell'} \right\rangle = \delta_{\ell\ell'}. \qquad (36)$$

This definition is allowed because here we are not calculating the absolute value of the form factors. The appropriate normalization will be determined in future papers in which we will give several kinds of form factors. This way of solving (31) is unique. Although we will not use in this paper, one can obtain $\Lambda_+ \otimes \Psi_2^\ell$ as

$$\Lambda_+ \otimes \psi_2^\ell = \sum_{\ell'} c_{2-}^{\ell\ell'} \Psi_{\ell'}^-, \qquad (37)$$

with the coefficients,

$$c_{2-}^{\ell\ell'} = \frac{1}{2m_Q} \left\langle \Psi_{\ell'}^- \middle| \left(\left(H_0 - E_0^\ell\right) \Lambda_+ \otimes \psi_1^\ell + H_1 \Lambda_+ \otimes \psi_0^\ell\right) \right\rangle. \qquad (38)$$

IV. NUMERICAL ANALYSIS

In this section, we give a numerical analysis of the calculations obtained by applying our formulation, i.e., perturbatively expanding the Hamiltonian given by Eq. (10) in $1/m_Q$ and computing all the matrix elements among eigenfunctions, $\Psi_\ell^\pm$ in terms of this set of eigen. In order to solve Eq. (24), we have to numerically obtain a radial part of the wave function, $\Psi_\ell^+ = \begin{pmatrix} 0 & \Psi_{jm}^k \end{pmatrix}$, given by

$$\Psi_{jm}^k(\vec{r}) = \frac{1}{r} \begin{pmatrix} u_k(r) \\ -i\, v_k(r)\, (\vec{\sigma} \cdot \vec{n}) \end{pmatrix} y_{jm}^k(\Omega),$$

some properties of which are descibed in the paper[8]. As described in the same paper, the Schrödinger equation is reduced into,

$$\begin{pmatrix} m_q + S + V & -\partial_r + \frac{k}{r} \\ \partial_r + \frac{k}{r} & -m_q - S + V \end{pmatrix} \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix} = E_k^0 \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix},$$

which is solved numerically by taking into account the asymptotic behaviors at both $r \to 0$ and $r \to \infty$ and their forms are given by

$$u_k(r),\ v_k(r) \sim w_k(r)\, r^{\gamma} \exp\left(-(m_q+b)\, r - \frac{1}{2}\left(\frac{r}{a}\right)^2\right), \quad (39)$$

where

$$\gamma = \sqrt{k^2 - \left(\frac{4\alpha_s}{3}\right)^2} \quad (40)$$

and $w_k\,(r)$ is a finite series of a polynomial of r

$$w_k(r) = \sum_{i=0}^{N} a_i^k\, r^i, \quad (41)$$

which takes different coefficients, a_i^k, for $u_k\,(r)$ and $v_k\,(r)$. In our case, since the potential includes a scalar term we cannot annalytically solve the above reduced Schrödinger equation, Eq. (5).

To determine the parameters, α_s, a, and b, appearing the potentials given by Eq. (11), we have calculated the chi square defined by

$$\chi^2 = \frac{(M_D - E_D)^2}{\sigma_D^2} + \frac{(M_{D^*} - E_{D^*})^2}{\sigma_{D^*}^2}, \quad (42)$$

where $M_{D,\ D^*}$ and $E_{D,\ D^*}$ are the observed and calculated masses of D and D^*, respectively and $\sigma_{D,\ D^*}$ are the experimental errors for each meson mass. Masses, $M_{D,\ D^*}$, are averaged over charges since we have not taken into account the electromagnetic interaction. We have adopted the values for these parameters which give the most minimum value of χ^2, which is listed in Table II. Then using the observed values of $M_{Ds,\ Ds^*}$, the s quark mass, m_s, is determinedand finally using the observed value of M_B, the b quark mass, m_b, is determined, which are given also in the Table II togother with the observed input mass vlues listed in the Table I.

TABLE I. Input values to determine parameters

$m_q = m_u = m_d$ (GeV)	M_D (GeV)	M_{D^*} (GeV)	M_{Ds} (GeV)	M_{Ds}^* (GeV)	M_B (GeV)
0.01	1.867	2.008	1.969	2.110	5.279

TABLE II. Most optimal values of parameters determined by the least chi square method.

parameters	α_s	a (GeV^{-1})	b (GeV)	m_c (GeV)	m_s (GeV)	m_b (GeV)	χ^2
(1)	0.4182	2.3661	0.07526	1.445	0.136(7.69$\times 10^{-6}$)	4.834	7.1$\times 10^{-7}$
(2)	0.3274	2.1284	-0.08689	1.375	0.172(1.75$\times 10^{-6}$)	4.795	2.4$\times 10^{-5}$

We have obtained two choice of parameters for the least chi square between which we have chosen the one with the parameter b positive since this set of parameters give the good value for m_c used in the references. The masses calculated and listed in this paper below are all based on the choice (1) in the TableII.

Two states, pseudoscalar ($D(0^-)$) and vector ($D(1^-)$), are degenerate at the lowest order in $1/m_Q$ since the eigenvalue E_0^k for these states depends on the same quantum number k=-1, which are split via the heavy quark spin interaction terms, like $-V'(\vec{\alpha}_q\cdot\vec{\Sigma}_Q \times \vec{n})$ in $\Lambda^- H_1 \Lambda^-$and all terms in $\Lambda^- H_1 \Lambda^+$. Similar resolution of the degeneracy among the states with the same value of k occurs via the same interaction terms. These calculated masses together with others (D_s, D_s^*, B, B^*, B_s, B_s^*, etc.) are listed in the Tables

TABLE III. D meson mass spectrum

state (J^P)	k	j	zeroth ($m_Q + m_Q$)	first / ($m_Q + m_Q$)	M_{calc}	M_{obs}
$^1S_0$ (0^-)	-1	0	1.900	-7.73 ($\times 10^{-2}$)	1.867	1.867
$^3S_1$ (1^-)	-1	1		5.69	2.008	2.008
$^3P_0$ (0^+)	1	0	2.217	-2.02	2.172	-
$^3P_1$ (1^+)	1	1		9.56	2.428	2.428(?)
$^1P_1$ (1^+)	-2	1	2.227	6.49	2.371	-
$^3P_2$ (2^+)	-2	2		8.38	2.413	2.457
$^3D_1$ (1^-)	2	1	2.480	7.40	2.664	-
$^3D_2$ (2^-)	2	2		8.85	2.700	-

TABLE IV. D_s meson mass spectrum

state (J^P)	k	j	zeroth ($m_Q + m_Q$)	first / ($m_Q + m_Q$)	M_{calc}	M_{obs}
$^1S_0$ (0^-)	-1	0	1.977	-0.371 ($\times 10^{-2}$)	1.970	1.969
$^3S_1$ (1^-)	-1	1		6.51	2.106	2.110(?)
$^3P_0$ (0^+)	1	0	2.293	-1.04	2.269	-
$^3P_1$ (1^+)	1	1		11.3	2.552	2.535
$^1P_1$ (1^+)	-2	1	2.312	6.93	2.473	-
$^3P_2$ (2^+)	-2	2		8.87	2.517	-
$^3D_1$ (1^-)	2	1	2.562	7.90	2.764	-
$^3D_2$ (2^-)	2	2		9.39	2.803	-

TABLE V. B meson mass spectrum

state (J^P)	k	j	zeroth ($m_Q + m_Q$)	first / ($m_Q + m_Q$)	M_{calc}	M_{obs}
$^1S_0$ (0^-)	-1	0	5.289	-0.186 ($\times 10^{-2}$)	5.279	5.279
$^3S_1$ (1^-)	-1	1		0.611	5.321	5.325
$^3P_0$ (0^+)	1	0	5.606	-0.238	5.592	-
$^3P_1$ (1^+)	1	1		1.12	5.669	-
$^1P_1$ (1^+)	-2	1	5.616	0.769	5.659	-
$^3P_2$ (2^+)	-2	2		0.993	5.672	-
$^3D_1$ (1^-)	2	1	5.869	0.935	5.924	-
$^3D_2$ (2^-)	2	2		1.12	5.935	-

TABLE VI. B_S meson mass spectrum

state (J^P)	k	j	zeroth $(m_0 + m_Q)$	first / $(m_0 + m_Q)$	M calc	M obs
$^1S_0\,(0^-)$	-1	0	5.366	-0.0409 ($\times 10^{-2}$)	5.364	5.375
$^3S_1\,(1^-)$	-1	1		0.717	5.405	-
$^3P_0\,(0^+)$	1	0	5.682	-0.125	5.675	-
$^3P_1\,(1^+)$	1	1		1.36	5.759	-
$^1P_1\,(1^+)$	-2	1	5.702	0.840	5.749	-
$^3P_2\,(2^+)$	-2	2		1.07	5.763	-
$^3D_1\,(1^-)$	2	1	5.951	1.02	6.012	-
$^3D_2\,(2^-)$	2	2		1.21	6.023	-

V. COMMENTS AND DISCUSSIONS

In this paper, we have calculated heavy meson masses of D, D_S, B, and B_S based on the formulation proposed before,[4] which develops the perturbation of a potential theory in terms of inverse power of a heavy quark mass. The first order calculation gives a good agreement with the experimental data although the seond one does not.

We have also found a new symmetry already mentioned in the paper [4] and realized by the operator,

$$-\beta_q\left(\vec{\Sigma}_q\cdot\vec{\ell}+1\right),$$

which is always present when one considers a centrally symmetric potential model for two particles or when one takes a rest frame limit of a general relativistic form of the wave function and is related to a light quark spin structure, i.e., $y^k_{jm}(\Omega)$. That is, this is quite a general symmetry, not a special feature peculiar to the potential model.

One can easily see degeneracy among the lowest lying pseudoscalar and vector states as follows. Define

$$|P\rangle = U_c^{-1}(0 \quad \Psi^{-1}_{00}), \quad |V,\lambda\rangle = U_c^{-1}(0 \quad \Psi^{-1}_{1\lambda}), \tag{43}$$

where Ψ^k_{jm} is an eigenfunction obtained in the last chapter, The explicit forms of these wave functions are given in the paper[8] and the quantum number k can take only $\pm j$, or $\pm(j+1)$. Assigning these states to D mesons, one can have

$$|P\rangle = |D^{\pm}\rangle,\ \text{or}\ |D^0\rangle, \quad |V,\lambda\rangle = |D^*\rangle. \tag{44}$$

Since these states have the same quantum number k=-1, these have the same masses as well as the same wave functions up to the zeroth order calculation in $1/m_Q$. That is, the degeneracy among these states is simply the result of properties of the eigenvalue equation. Higher order corrections can be solved by developing perturbation of energy and wave function for each state in terms of m_q/m_Q as given by Eqs.(19, 20)

$$\tilde{E} \equiv E - m_Q = E^\ell_0 + E^\ell_1 + E^\ell_2 + \dots,$$
$$\psi_{FWT} = \psi^\ell_0 + \psi^\ell_1 + \psi^\ell_2 + \dots,$$

Next we would like to discuss qualitative features of form factors/ Isgure-Wise functions. Let us think about to calculate form factors for semileptonic decay of B meson into D. Taking a simple form for the lowest lying wave function both for B and D as

$$\psi^{1S} \sim e^{-b^2 r^2/2},$$

where a parameter B is determined by a variational principle, $\delta(\Psi^{1S\dagger} H \Psi^{1S}) = 0$. Then form factors are given by

$$F(q^2) \sim \exp\left[\text{const.}\ \tilde{E}^2 (q^2 - q^2_{\max})\right],\ \text{or}\quad \xi(\omega) \sim \exp\left[\text{const.}\ \tilde{E}^2(\omega - 1)\right],$$

where

$$q^2 = (p_B - p_D)^2, \quad \omega = v_B\cdot v_D, \quad q^2_{\max} = (m_B - m_D)^2 \leftrightarrow \omega_{\max} = 1,$$

with $v^{\mu}_{B,D}$ being four-velocity of B and/or D meson. This means behavior of form factors strongly depends on an eigenvalue, $\tilde{E} = E\text{-}m_Q$ of the eigenvalue equation, which is often called "inertia" parameter $\bar{\Lambda}_q$. This quantity $\tilde{E}$ does not depend on any heavy quark properties at the zeroth order. This result also means that the slope at the origin of the Isgur-Wise function includes the term proportional to $\tilde{E}^2$. The constant term (-1/4 like the Bjorken limit[9]) for this slope should be given by a kinematical factor multiplied with the above expression.

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REFERENCES

[1]) M. Voloshin and M. Shifman, *Sov. J. Nucl. Phys.* **45**, 292 (1987) and 47, 511 (1988);
N. Isgur and M. Wise, *Phys. Lett.* **B232**, 113 (1989) and **B237**, 527 (1990);
E. Eichten and B. Hill, *Phys. Lett.* **B234**, 511 (1990);
B. Grinstein, *Nucl. Phys.* **B339**, 253 (1990);
H. Georgi, *Phys. Lett.* **B240**, 447 (1990);
A. Falk, H. Georgi, B. Grinstein and M. Wise, *Nucl. Phys.* **B343**, 1 (1990).

[2]) M. E. Luke, *Phys. Lett.* **B252**, 447 (1990);
J. G. Körner and G. Thompson, *Phys. Lett.* **B264**, 185 (1991);
T. Mannel, W. Roberts and Z. Ryzak, *Nucl. Phys.*
368, 204 (1992);
S. Balk, J. G. Körner and D. Pirjol, *Nucl. Phys.* **B428**, 499 (1994);

[3]) See for review,
N. Isgur and M. *B. Wise*, B Decays, (ed. S. Stone,
World Scientific, Singapore, 1992)

[4]) T. Matsuki, Mod. *Phys. Letters* **A11**, 257 (1996).

[5]) L. L. Foldy and S. A. Wouthuysen, *Phys. Rev.* **78**, 29 (1950);
S. Tani, *Prog. Theor.* Phys. **6**, 267 (1951).

[6]) J. Morishita, M. Kawaguchi and T. Morii, *Phys. Rev.* **37**, 159 (1988).

[7]) S. N. Mukherjee, R. Nag, S. Sanyal, T. Morii, J. Morishita and M. Tsuge, *Phys. Rep.* **231**, 201 (1993).

[8]) A talk given by T. Matsuki, *Proceedings of the IV*
International Symposium on Weak and Electromagnetic Interactions in Nuclei (ed. H. Ejiri, T. Kishimoto, and T. Sato, World Scientific, Singapore, 1995), p.536; T. Matsuki and T. Morii, preprint in preparation.

[9]) J.D. Bjorken, *Invited talk given at Les Recontre de la Valle D'Aoste*, La Thuile, Italy, 1990;
See also the paper by J.D. Bjorken, I. David, and J. Taron in the reference Ref. 1 and M. Suzuki, *Nucl. Phys.* **B258**, 553 (1985).

Spectroscopy of Heavy Mesons Expanded in $1/m_Q$

質量分の１展開した重粒子のスペクトル（英文）

松木　孝幸，渡辺　丕俊，井上　俊哉

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ワークステーション上のＣ言語プログラミング用のライブラリを開発したのであるが，その応用として「質量分の１展開した重粒子のスペクトル」を計算した．いわゆるフォルディー変換を２体の束縛粒子の波動関数に適用し，シュレーディンガー方程式を質量分の１展開して解いた．その時に適当なポテンシャルを仮定し，それらに含まれるパラメターを決定し，他の重い粒子の質量値を予言した．本文中の計算結果から見られるようにモデルとして用いたポテンシャルはよく実験値を再現している．